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Intersecting non-SUSY p-brane with chargeless 0-brane as black p-brane

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Abstract: Unlike BPS p-brane, non-supersymmetric (non-susy) p-brane could be either charged or chargeless. As envisaged in [hep-th/0503007], we construct an intersecting non-susy p-brane with chargeless non-susy q-brane by taking T-dualities along the delocalized directions of the non-susy q-brane solution delocalized in $(p-q)$ transverse directions (where $p \geq q$). In general these solutions are characterized by four independent parameters. We show that when $q = 0$ the intersecting charged as well as chargeless non-susy p-brane with chargeless 0-brane can be mapped by a coordinate transformation to black p-brane when two of the four parameters characterizing the solution take some special values. For definiteness we restrict our discussion to space-time dimensions $d = 10$. We observe that parameters characterizing the black brane and the related dynamics are in general in a different branch of the parameter space from those describing the brane-antibrane annihilation process. We demonstrate this in the two examples, namely, the non-susy D0-brane and the intersecting non-susy D4 and D0-branes, where the solutions with the explicit microscopic descriptions are known.

Keywords: p-branes, Black Holes in String Theory, D-branes, Tachyon Condensation.

Contents

1. Introduction

The static, non-susy and asymptotically flat p-brane solutions[1] having isometries ISO$(p,1)$ $\times$ SO$(d-p-1)$ of type II supergravities in arbitrary space-time dimensions (d) are given in ref. [1].[2] Unlike the BPS p-branes characterized by a single unknown parameter, these solutions are characterized by three unknown parameters and could be either charged or chargeless with respect to a $(p+1)$-form gauge field. It is well-known that these non-susy p-branes have a natural interpretation as coincident p-brane-anti-p-brane (or non-BPS p-brane) [4 − 6]. Since there is an open string tachyon on their world-volume, these systems are unstable [7]. The three parameters in the supergravity solutions then have physical meanings in terms of number of branes, number of antibranes and the tachyon parameter. The explicit semi-empirical relations of the supergravity parameters in terms of the microscopic physical parameters are given in ref. [5, 6], so that it correctly captures the picture of tachyon condensation for these systems.

On the other hand, there is another class of non-susy p-brane solutions in type II supergravities in the form of black p-branes with isometries R $\times$ ISO(p) $\times$ SO$(d-p-1)$ [8 − 10]. Unlike the previous solutions which can have naked singularities for certain choices of parameters, the black p-branes always have regular horizons and are characterized by two parameters corresponding to the mass and the charge of the black branes. In the literature these two classes are referred to as type 1 and type 2 solutions [11].

However, in contrary to the previous claim [11], we will argue that there is only one class of non-susy p-brane solutions, namely, the type 1. We will restrict our discussion to $d = 10$ for definiteness. The black brane solutions, i.e. the type 2, with the above isometries belong to a more general non-susy p-brane solutions of type 1 than the one mentioned in the first

[1]We use the terminology non-susy p-brane to represent generically either the p-brane-anti$p$-brane system or the non-BPS p-branes.

[2]See [2, 3] for earlier works on non-susy p-branes.

paragraph. We will show that the black p-brane solutions are nothing but the intersecting[3] non-susy p-branes with the chargeless 0-branes expressed in a different coordinate system, when two of the four parameters characterizing the latter solutions take some special values. In order to show that, we will first construct an intersecting non-susy brane solutions where the constituent branes are the charged non-susy p-brane and the chargeless non-susy q-brane. The explicit solutions when both the non-susy branes in the intersecting solution are charged are known only when $p-q=4$ [6] (or the simple case $p-q=0$ [1]). Here we are interested in solutions when non-susy q-brane is chargeless. As mentioned in ref. [13], these solutions can be obtained from the non-susy q-brane solutions delocalized in $(p-q)$ transverse directions and then applying T-dualities along those delocalized directions. We then make all the T-dual directions isometric resulting in an intersecting non-susy p-brane with chargeless non-susy q-brane solutions (where $p \geq q$). These solutions belong to type 1 and as we will show are characterized by four independent parameters. Next we show that for $q=0$, these solutions can be mapped to the black p-brane solutions mentioned above as type 2, by a coordinate transformation when two of the four parameters take some special values. This possibility is expected since the presence of 0-brane breaks the isometry of non-susy p-brane from $\mathrm{ISO}(p,1) \times \mathrm{SO}(9-p)$ to $\mathrm{R} \times \mathrm{ISO}(p) \times \mathrm{SO}(9-p)$ and the black brane has two parameters not four as in the non-susy brane solutions.

The above observation, however, gives rise to a puzzle. As we mentioned for the non-susy p-branes, the intersecting non-susy p-brane with a non-susy q-brane also is an unstable system which is manifested by the presence of open string tachyon on their world-volume. Since the parameters of these solutions are related to the microscopic physical parameters (this is explicitly known for $p-q=0,4$) of the intersecting brane-antibrane system [5, 6], the SUGRA parameters change their values as the brane-antibrane annihilation process (or the tachyon condensation) takes place and finally reach their end values associated with the BPS configuration. However, in this process we never encounter the formation of a regular horizon for these systems. Then how for $q=0$ and certain values of the parameters we get a black brane solutions? As we will see, there actually exists an additional disjoint branch[4] in the parameter space which has been neglected so far. This branch can give rise to the regular horizon formation for the black brane where the underlying dynamics is governed by the possible closed string tachyon condensation as discussed by Horowitz in [14, 15] and evolves into a non-susy "bubble of nothing". In this branch one of the parameters of the solution becomes unbounded in general and as such makes the solution complex [1], signalling a possible phase transition and this is consistent with our above observation. We will give some evidence for this. However, in this paper we limit ourselves to finding the values of the parameters required for the black brane in general only from the second branch and explain some related issues leaving a detailed study along with other related issues to a future publication. Also our discussion for providing evidences is limited only to the case of non-susy D0-branes (which includes D0/$\bar{\mathrm{D}}$0 and non-BPS D0-branes) and intersecting non-susy D4-branes with chargeless non-susy D0-branes. The reason for choosing these two

[3] A particular kind of intersecting non-susy brane solutions were constructed previously in [12, 6].

[4] Note, however, that for the complete chargeless case there is a meet point at the initial tachyon value $T=0$, i.e., at the top of the tachyon potential as will be shown in the examples given later in this paper.

systems is that only for these two cases we have the explicit relations between the SUGRA parameters and the microscopic physical parameters describing correctly the process of open string tachyon condensation [5, 6].

This paper is organized as follows. In section 2, we give the four parameter solution of intersecting non-susy p-brane with chargeless non-susy q-brane. In section 3, we show that for $q = 0$, these solutions get mapped to black p-brane when two of the four parameters take some specific values. In section 4, we use the microscopic descriptions of D0/$\bar{\text{D}}$0 and also the intersecting D4/$\bar{\text{D}}$4 and chargeless D0/$\bar{\text{D}}$0 system to show that the parameter space characterizing the usual black branes and the related dynamics is in a different branch from that describing the brane-antibrane annihilation process ending up with a BPS or Minkowski configuration. Our conclusion is presented in section 5.

2. Intersecting brane solution

In this section we will give the construction of intersecting non-susy p-brane with chargeless non-susy q-brane, where, $p \geq q$. For this purpose we will start with the non-susy q-brane solution delocalized in $(p-q)$ transverse directions given in ref. [16]. The application of T-duality along each of the $(p-q)$ delocalized directions successively will give an anisotropic intersecting p brane with chargeless q-brane.[5] Then equating all the parameters associated with the various delocalized directions we get the isotropic, intersecting p-brane with chargeless q-brane. For $p - q = 4$ the intersecting solutions, when both the non-susy p-brane and the non-susy q-brane are charged, are known explicitly [6] and we will compare these solutions with the ones we obtain here. To begin let us write below the non-susy q-brane solution delocalized in $(p-q)$ transverse directions given in eq. (2.5) of ref. [16] as,

$$\begin{aligned} ds^2 &= F^{\frac{q+1}{8}} (H\tilde{H})^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}}\right)^{(-2\sum_{i=2}^{p-q+1}\delta_i)/(7-p)} \left(dr^2 + r^2 d\Omega_{8-p}^2\right) \\ &\quad + F^{-\frac{7-q}{8}} \left(-dt^2 + \sum_{i=1}^{q} (dx^i)^2\right) + F^{\frac{q+1}{8}} \sum_{i=2}^{p-q+1} \left(\frac{H}{\tilde{H}}\right)^{2\delta_i} (dx^{q+i-1})^2 \\ e^{2\phi} &= F^{-a} \left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[8-q]} = b\ \mathrm{Vol}(\Omega_{8-p}) \wedge dx^{q+1} \ldots \wedge dx^p \end{aligned} \tag{2.1}$$

Note that we have written the solution (2.5) of ref. [16] for $d = 10$ and we have replaced p there by q and q there by $(p-q)$ to represent non-susy q-brane delocalized in $(p-q)$

[5] Note that for BPS branes usually the same procedure does not give intersecting solutions and we get an isotropic, localized p-brane. The reason for getting a different solution here is that the non-susy solutions involve more parameters and break the isometry of the solution which can then be regarded as intersecting solutions as mentioned in ref. [13].

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directions. The various functions appearing in (2.1) are defined below,

$$
\begin{aligned}
F &= \left(\frac{H}{\tilde{H}}\right)^{\alpha} \cosh^2\theta - \left(\frac{\tilde{H}}{H}\right)^{\beta} \sinh^2\theta \\
H &= 1 + \frac{\omega^{7-p}}{r^{7-p}} \\
\tilde{H} &= 1 - \frac{\omega^{7-p}}{r^{7-p}}
\end{aligned} \tag{2.2}
$$

where H and $\tilde{H}$ are two harmonic functions and α, β, θ, δ_1, δ_2, $\ldots$, δ_{p-q+1}, and ω are $(p-q+5)$ integration constants and b is the charge parameter. However, there are three relations among the parameters given as,

$$
\begin{aligned}
&\alpha - \beta = a\delta_1 \\
&\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha - a\delta_1) + \frac{2\sum_{i>j=2}^{p-q+1}\delta_i\delta_j}{7-p} = \left(1 - \sum_{i=2}^{p-q+1}\delta_i^2\right)\frac{8-p}{7-p} \\
&b = (7-p)\omega^{7-p}(\alpha+\beta)\sinh 2\theta
\end{aligned} \tag{2.3}
$$

where $a = (q-3)/2$. So, using the above relations we can eliminate three of the parameters and the solution therefore has $(p-q+3)$ independent parameters. We will take T-duality along x^{q+1}, x^{q+2}, $\ldots$, x^p. For that we write the metric in (2.1) in the string frame as,

$$
\begin{aligned}
ds^2_{\rm str} &= e^{\phi/2} ds^2 \\
&= F^{\frac{1}{2}}(H\tilde{H})^{\frac{2}{7-p}}\left(\frac{H}{\tilde{H}}\right)^{(-2\sum_{i=2}^{p-q+1}\delta_i)/(7-p)+\frac{\delta_1}{2}} \left(dr^2 + r^2 d\Omega^2_{8-p}\right) \\
&\quad + F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}}\left(-dt^2 + \sum_{i=1}^{q}(dx^i)^2\right) + F^{\frac{1}{2}}\sum_{i=2}^{p-q+1}\left(\frac{H}{\tilde{H}}\right)^{2\delta_i + \frac{\delta_1}{2}}(dx^{q+i-1})^2
\end{aligned} \tag{2.4}
$$

Now when we apply T-dualities [17, 18] successively along the delocalized directions the dilaton changes as,

$$
e^{2\tilde{\phi}} = F^{\frac{3-p}{2}}\prod_{i=2}^{p-q+1}\left(\frac{H}{\tilde{H}}\right)^{-2\delta_i}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}(4-p+q)} \tag{2.5}
$$

T-dualities will not change the various components of the string metric in (2.4) except the ones $g_{q+1,q+1}$, $g_{q+2,q+2}$, $\ldots$, $g_{p,p}$. Let us write below the changed components,

$$
\begin{aligned}
\tilde{g}^{\rm str}_{q+1,q+1} &= F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{-2\delta_2 - \frac{\delta_1}{2}} \\
\tilde{g}^{\rm str}_{q+2,q+2} &= F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{-2\delta_3 - \frac{\delta_1}{2}} \\
&\vdots \\
\tilde{g}^{\rm str}_{p,p} &= F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{-2\delta_{p-q+1} - \frac{\delta_1}{2}}
\end{aligned} \tag{2.6}
$$

Now we rewrite the full T-dual solution in Einstein frame as

$$
\begin{aligned}
d\tilde{s}^2 &= e^{-\tilde{\phi}/2} d\tilde{s}^2_{\rm str}\\
&= F^{\frac{p+1}{8}} (H\tilde{H})^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{(p-q)\delta_1}{8}+\frac{(3-p)\sum_{i=2}^{p-q+1}\delta_i}{2(7-p)}} \left(dr^2 + r^2 d\Omega^2_{8-p}\right)\\
&+F^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{\frac{(p-q)\delta_1}{8}+\sum_{i=2}^{p-q+1}\frac{\delta_i}{2}} \left(-dt^2 + \sum_{i=1}^{q}(dx^i)^2\right)\\
&+F^{-\frac{7-p}{8}} \sum_{i=2}^{p-q+1} \left(\frac{H}{\tilde{H}}\right)^{-\delta_1+\frac{\delta_1}{8}(p-q)-\frac{3\delta_i}{2}+\sum_{j(\neq i)=2}^{p-q+1}\frac{\delta_j}{2}} (dx^{q+i-1})^2\\
e^{2\tilde{\phi}} &= F^{\frac{3-p}{2}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}(4-p+q)-\sum_{i=2}^{p-q+1}2\delta_i}\\
F_{[8-p]} &= b{\rm Vol}(\Omega_{8-p}) \qquad (2.7)
\end{aligned}
$$

where $d\tilde{s}^2_{\rm str}$ is the metric in (2.4) with the $g_{q+1,q+1}$, $g_{q+2,q+2}$, $\ldots$, $g_{p,p}$ replaced by the ones given in eq. (2.6). Now we make all the delocalized directions isotropic by setting $\delta_2 = \delta_3 = \cdots = \delta_{p-q+1}$. The solution (2.7) therefore takes the form,

$$
\begin{aligned}
ds^2 &= F^{\frac{p+1}{8}} (H\tilde{H})^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{(p-q)\delta_1}{8}+\frac{(p-q)(3-p)\delta_2}{2(7-p)}} \left(dr^2 + r^2 d\Omega^2_{8-p}\right)\\
&+F^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{\frac{(p-q)\delta_1}{8}+\frac{(p-q)\delta_2}{2}} \left(-dt^2 + \sum_{i=1}^{q}(dx^i)^2\right)\\
&+F^{-\frac{7-p}{8}} \sum_{j=q+1}^{p} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{8}(p-q-8)+\frac{(p-q-4)\delta_2}{2}} (dx^j)^2\\
e^{2\phi} &= F^{\frac{3-p}{2}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}(4-p+q)-2(p-q)\delta_2}\\
F_{[8-p]} &= b{\rm Vol}(\Omega_{8-p}) \qquad (2.8)
\end{aligned}
$$

Note that we have removed 'tilde' from $d\tilde{s}^2$ and $e^{2\tilde{\phi}}$ for brevity. The parameter relations (2.3) now take the forms,

$$
\begin{aligned}
\alpha - \beta &= a\delta_1\\
\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha - a\delta_1) + \frac{(p-q)(p-q-1)}{7-p}\delta_2^2 &= \left(1-(p-q)\delta_2^2\right)\frac{8-p}{7-p}\\
b &= (7-p)\omega^{7-p}(\alpha+\beta)\sinh 2\theta \qquad (2.9)
\end{aligned}
$$

where the functions F and H, $\tilde{H}$ are as given in eq. (2.2). It is clear from the above that the metric in (2.8) has an isometry ISO$(1,q)$ $\times$ SO$(p-q)$ $\times$ SO$(9-p)$ and therefore the solution can be interpreted as intersecting non-susy p-brane with non-susy q-brane. It is clear that since we have $F_{[8-p]}$ which is in general non-zero so, the non-susy p-brane is charged under it, but since there is no $F_{[8-q]}$ in the solution, the charge of the non-susy

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q-brane is zero. The solution is characterized by four unknown independent parameters ω, θ, δ_1 and δ_2. So, (2.8) represents the four parameter solution of intersecting non-susy p-brane with chargeless non-susy q-brane. Although we have concluded that (2.8) represents the required solution from the symmetry of the metric, in order to convince ourselves we will compare it with the intersecting brane solutions obtained earlier for $p-q=4$ [6] since this is only case where explicit solution is known. For this purpose we will define another function $\hat{F}$ by,

$$F = \left(\frac{H}{\tilde{H}}\right)^{\alpha} \cosh^2\theta - \left(\frac{\tilde{H}}{H}\right)^{\beta} \sinh^2\theta \;= \hat{F}\left(\frac{H}{\tilde{H}}\right)^{\tilde{\alpha}} \tag{2.10}$$

where

$$\hat{F} = \left(\frac{H}{\tilde{H}}\right)^{\alpha_1} \cosh^2\theta - \left(\frac{\tilde{H}}{H}\right)^{\beta_1} \sinh^2\theta \tag{2.11}$$

where we have defined $\alpha_1 + \tilde{\alpha} = \alpha$ and $\beta_1 - \tilde{\alpha} = \beta$. Now it can be checked that by defining the parameters as,

$$\begin{aligned}
\tilde{\alpha} &= \frac{7-p}{7-q}\alpha_2 - \frac{p-q}{7-q}\delta \\
\delta_2 &= -\frac{7-p}{2(7-q)}(\alpha_2+\delta) \\
\delta_1 &= -\frac{p-q}{7-q}\alpha_2 + \frac{7-p}{7-q}\delta
\end{aligned} \tag{2.12}$$

we can recast the solution (2.8) as follows,

$$\begin{aligned}
ds^2 &= \hat{F}^{\frac{p+1}{8}} \left(H\tilde{H}\right)^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{q+1}{8}\alpha_2} \left(dr^2 + r^2 d\Omega^2_{8-p}\right) \\
&\quad + \hat{F}^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{7-q}{8}\alpha_2} \left(-dt^2 + \sum_{i=1}^{q}(dx^i)^2\right) + \hat{F}^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{\frac{q+1}{8}\alpha_2} \sum_{j=q+1}^{p}(dx^j)^2 \\
e^{2\phi} &= \hat{F}^{\frac{3-p}{2}} \left(\frac{H}{\tilde{H}}\right)^{\frac{3-q}{2}\alpha_2 + 2\delta} \\
F_{[8-p]} &= b\,\mathrm{Vol}(\Omega_{8-p})
\end{aligned} \tag{2.13}$$

The parameter relations as obtained from (2.9) now takes the form,

$$\begin{aligned}
\alpha_1 - \beta_1 &= \left(\frac{p-q}{2} - 2\right)\alpha_2 + \frac{p-3}{2}\delta \\
b &= (7-p)\omega^{7-p}(\alpha_1+\beta_1)\sinh 2\theta \\
\frac{1}{2}\left(\delta - \frac{p-3}{4}\alpha_1 - \frac{q-3}{4}\alpha_2\right)^2 &= \frac{8-p}{7-p} - \frac{(p+1)(7-p)}{32}\alpha_1^2 - \frac{(q+1)(7-q)}{32}\alpha_2^2 \\
&\quad - \frac{(q+1)(7-p)}{16}\alpha_1\alpha_2
\end{aligned} \tag{2.14}$$

The solution (2.13) is characterized by four independent parameters ω, θ, δ and α_2. Now comparing (2.13) with the intersecting non-susy Dp/D$(p-4)$ solution given in eq. (2.1) of ref. [6], we find that they exactly match for $q = p-4$ and when $\theta_1 = \theta$, $\theta_2 = 0$ indicating that the non-susy D$(p-4)$ brane is chargeless. The parameter relation (2.14) also match with the parameter relations given there. Thus comparing the explicit solution of intersecting non-susy Dp/D$(p-4)$, we conclude that (2.8) indeed represents intersecting non-susy p-brane with chargeless non-susy q-brane solution.

3. Intersecting solution and black p-brane

In this section we will show that when $q = 0$, either of the intersecting solution (2.8) or (2.13) can be mapped to the standard black p-brane solution [8–10] by a coordinate transformation for certain choice of two of the four parameters characterizing the solution. For this purpose let us first write the solution (2.13) for general p and $q = 0$ as follows,

$$\begin{aligned}
ds^2 &= \hat{F}^{\frac{p+1}{8}} \left(H\tilde{H}\right)^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{1}{8}\alpha_2} \left(dr^2 + r^2 d\Omega_{8-p}^2\right) \\
&\quad - \hat{F}^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{7}{8}\alpha_2} dt^2 + \hat{F}^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{\frac{1}{8}\alpha_2} \sum_{j=1}^{p} (dx^j)^2 \\
e^{2\phi} &= \hat{F}^{\frac{3-p}{2}} \left(\frac{H}{\tilde{H}}\right)^{\frac{3}{2}\alpha_2 + 2\delta} \\
F_{[8-p]} &= b \mathrm{Vol}(\Omega_{8-p})
\end{aligned} \tag{3.1}$$

where the parameters satisfy

$$\begin{aligned}
\alpha_1 - \beta_1 &= \left(\frac{p}{2} - 2\right)\alpha_2 + \frac{p-3}{2}\delta \\
b &= (7-p)\omega^{7-p}(\alpha_1 + \beta_1)\sinh 2\theta \\
\frac{1}{2}\left(\delta - \frac{p-3}{4}\alpha_1 + \frac{3}{4}\alpha_2\right)^2 &= \frac{8-p}{7-p} - \frac{(p+1)(7-p)}{32}\alpha_1^2 - \frac{7}{32}\alpha_2^2 - \frac{7-p}{16}\alpha_1\alpha_2
\end{aligned} \tag{3.2}$$

Let us now make a coordinate transformation from r to ρ as follows,

$$r = \rho \left(\frac{1+\sqrt{f}}{2}\right)^{\frac{2}{7-p}} \tag{3.3}$$

where we have defined,

$$f = 1 - \frac{4\omega^{7-p}}{\rho^{7-p}} \equiv 1 - \frac{\rho_0^{7-p}}{\rho^{7-p}} \tag{3.4}$$

From eq. (3.3) we find

$$\begin{aligned}
H &= \frac{2}{1+\sqrt{f}} \\
\tilde{H} &= \frac{2\sqrt{f}}{1+\sqrt{f}}
\end{aligned} \tag{3.5}$$

where H and $\tilde{H}$ are as given before in eq. (2.2). From here we also obtain

$$\frac{H}{\tilde{H}} = f^{-\frac{1}{2}} \tag{3.6}$$

Now from the relation (2.11) we write

$$\hat{F} = \left[\cosh^2\theta - \sinh^2\theta\left(\frac{\tilde{H}}{H}\right)^{\alpha_1+\beta_1}\right]\left(\frac{H}{\tilde{H}}\right)^{\alpha_1} \tag{3.7}$$

When $\alpha_1 + \beta_1 = 2$, the relation (3.7) reduces to

$$\begin{aligned}\hat{F} &= \left(\cosh^2\theta - \sinh^2\theta f\right) f^{-\frac{\alpha_1}{2}}\\ &= \left(1 + \frac{\rho_0^{7-p}\sinh^2\theta}{\rho^{7-p}}\right) f^{-\frac{\alpha_1}{2}}\\ &= \bar{H} f^{-\frac{\alpha_1}{2}}\end{aligned} \tag{3.8}$$

where we have defined $\bar{H} = 1 + \sinh^2\theta\rho_0^{7-p}/\rho^{7-p}$. Also using

$$\begin{aligned}H\tilde{H} &= \frac{4\sqrt{f}}{1+f+2\sqrt{f}}\\ dr &= \frac{1}{\sqrt{f}}\left\{\frac{1+\sqrt{f}}{2}\right\}^{\frac{2}{7-p}} d\rho\end{aligned} \tag{3.9}$$

we find that the metric in (3.1) would reduce to the following black p-brane form,

$$ds^2 = \bar{H}^{\frac{p+1}{8}}\left(\frac{1}{f}d\rho^2 + \rho^2 d\Omega_{8-p}^2\right) + \bar{H}^{-\frac{7-p}{8}}\left(-f dt^2 + \sum_{j=1}^{p}(dx^j)^2\right) \tag{3.10}$$

if the parameters α_1 and α_2 satisfy,

$$\alpha_1 = \frac{2}{7-p}, \qquad \text{and} \qquad \alpha_2 = 2. \tag{3.11}$$

Now from the parameter relation (3.2) we also obtain the value of δ as

$$\delta = -\frac{2(6-p)}{7-p} \tag{3.12}$$

and the so the dilaton in (3.1) reduces to

$$e^{2\phi} = \bar{H}^{\frac{3-p}{2}} \tag{3.13}$$

The charge relation in (3.2) becomes

$$b = 2(7-p)\omega^{7-p}\sinh 2\theta = \frac{1}{2}(7-p)\rho_0^{7-p}\sinh 2\theta \tag{3.14}$$

The eqs. (3.10), (3.13) along with (3.14) represent the known black p-brane solution [8–10]. Note that in obtaining the black p-brane solution we have used the intersecting solution (3.1) i.e. we have used (2.13) with general p and $q = 0$. The parameters in this solution are ω, θ, δ and α_2 (α_1 and β_1 are not independent). However, black p-brane solution could also be obtained using intersecting solution (2.8) where the unknown parameters are ω, θ,

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δ_1 and δ_2 (here α and β are not independent). Using (2.12) in (3.11) and (3.12) we can obtain the value of the parameters in solution (2.8) such that it represent the black p-brane solution. The values are,

$$\alpha + \beta = 2, \qquad \alpha = \frac{16}{7}, \qquad \beta = -\frac{2}{7}$$
$$\delta_1 = -\frac{12}{7}, \qquad \delta_2 = -\frac{1}{7} \tag{3.15}$$

Note that unlike in the previous case, the values of the parameters here are universal in the sense that they are independent of p and will be useful for our discussion of describing the two branches in the parameter space in the next section.

Before closing this section let us make a few comments on the solution in general.

- Note that the function F or $\hat{F}$ as defined in eq. (2.2) or eq. (2.11) has to be non-negative in general if the metric of the intersecting brane solution has to remain real. Now for general non-vanishing θ one can write $F = \left[\cosh^2\theta - \sinh^2\theta\left(\frac{\tilde{H}}{H}\right)^{\alpha+\beta}\right]\left(\frac{H}{\tilde{H}}\right)^{\alpha}$ and similarly for $\hat{F}$. This implies that $\alpha+\beta \geq 0$ (or similarly $\alpha_1 + \beta_1 \geq 0$), otherwise F can not be kept non-negative once r approaches ω. So, we need in general $\alpha+\beta \geq 0$, but for $\theta = 0$, we can have $\alpha + \beta < 0$ as well.

- Given our understanding that the black Dp-brane can be obtained from charged or chargeless non-susy Dp-brane with intersecting chargeless non-susy D0-brane only when $\alpha + \beta = 2 > 0$ and $\delta_1 = -12/7$, $\delta_2 = -1/7$, independent of the choice of other parameters, implies that black brane occurs only in the $\alpha+\beta \geq 0$-branch even though the other branch is allowed when $\theta = 0$.

- Let us now focus on $\alpha + \beta \geq 0$-branch. Even for this branch the black Dp-brane occurs only for particular choice of the other parameters $\alpha = 16/7$, $\delta_1 = -12/7$ and $\delta_2 = -1/7$. Then the natural question is what the other values of the parameters correspond to? Even for a given $\alpha + \beta \geq 0$, we can still have two branches of values for other parameters such as δ_1.

- In the $\alpha + \beta \geq 0$-branch the black Dp-brane has well-defined horizon but for other values of the parameters we have null or naked singularity for each configuration. One would expect that those with these singularities should be more unstable than the one with regular horizon because an observer at infinity will see the rapid annihilation process of the brane-antibrane for the system without a regular horizon than the one with a regular horizon. In the supergravity description the appearance of singularities in either case is due to supergravity approximation and in the full non-perturbative string language, we do not expect the singularities to form. The supergravity configurations only provide us an indication which process we should assign to.

- So, the above indicates that the parameter space of the (intersecting) non-susy brane configuration has various disjoint branches, which may imply a phase structure of the underlying dynamics. One branch gives rise to the brane-antibrane annihilation where the underlying dynamics is governed by the open string tachyon condensation [19]. Under this process the brane-antibrane configuration evolves into the final BPS brane or Minkowski vacuum depending on the initial charge of the system [5, 6]. In this branch we really do not have the condition of regular horizon formation to begin with except for the special complete chargeless case as mentioned earlier and to be discussed in the next section, and the solutions will have null or naked singularity (although this should be understood as the artifact of perturbative string theory). The other branch can give rise to the regular horizon formation and then the underlying dynamics is governed by possible closed string tachyon condensation [14, 15]. Under this process the black p-brane would evolve into bubble (so no singularity) according to Horowitz's prescription.

- Let us recall that the static non-susy p-brane or Dp/$\bar{\mathrm{D}}p$ brane configuration obtained in ref. [1], containing a parameter δ which was bounded from above for the solution to remain real. Right at the bounded value, the corresponding configuration has a null-singular horizon. We also noted that when the parameter exceeds the bound the solution becomes complex in general (except for a particular choice of θ where the configuration has periodically naked singularities), therefore, signalling a possible phase transition. This feature persists even for the intersecting non-susy brane solutions. In the next section we will demonstrate the similar characteristic, indicating the existence of two possible phases, for the two examples, namely for D0/$\bar{\mathrm{D}}$0 and intersecting D4/$\bar{\mathrm{D}}$4 with chargeless D0/$\bar{\mathrm{D}}$0 system where the explicit microscopic descriptions are known.

4. Dynamical structure of non-susy Dp-branes

In this section we give some evidence that the non-susy Dp-branes have two disjoint dynamical structures, one corresponding to the brane-antibrane annihilation process and the other related to the black p-brane dynamics. To illustrate the points we use the microscopic representation of the SUGRA parameters in terms of the physical parameters of the non-susy Dp-branes. These are known only for the D0/$\bar{\mathrm{D}}$0 system and the intersecting D4/$\bar{\mathrm{D}}$4 with chargeless D0/$\bar{\mathrm{D}}$0 system and we discuss the two cases separately below.

(a) D0/$\bar{\mathrm{D}}$0 system. The supergravity solution of D0/$\bar{\mathrm{D}}$0 system (when it is chargeless it can represent either the equal number of D0/$\bar{\mathrm{D}}$0 system or non-BPS D0-branes) can be obtained from eq. (2.8) by putting $p = q = 0$. This solution as discussed in ref. [1] is characterized by three independent SUGRA parameters ω, θ and δ_1. In ref. [5] we have related these parameters to the microscopic physical parameters namely the number of D0-branes (N), the number of $\bar{\mathrm{D}}$0-branes ($\bar{N}$) and the tachyon parameter (T) of the system. When we related the supergravity solution this way we assumed that the solution (2.8) represents one of a continuous family of the brane-antibrane configurations labeled by the tachyon parameter (T) in the brane-antibrane annihilation process. We also noted that the parameter δ_1 has to be bounded in order for this to be true. We noticed that when δ_1

exceeds that bound, the solution becomes complex and thereby signalling a possible phase transition in the system. In order for describing the brane-atibrane annihilation process through tachyon condensation such that the system evolves either to a BPS Dp-brane or Minkowski vacuum we found that the parameter δ_1 must be given as [5],

$$\delta_1^{(-)} = \frac{1}{2c_p}\frac{a}{|a|}\left\{|a|\sqrt{\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}} \right. \\ \left. -\sqrt{a^2\left(\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}\right) + 4\left(\frac{2(8-p)}{7-p}c_p^2 - \cos^2 T\right)}\right\} \tag{4.1}$$

where $a = (p-3)/2$ and c_p is an unknown constant depending on p but is bounded as

$$c_p \geq \frac{7-p}{4}\sqrt{\frac{p+1}{2(8-p)}} \tag{4.2}$$

in order for $\delta_1^{(-)}$ to be real. It is not difficult to check that the absolute value of $\delta_1^{(-)}$, i.e., $|\delta_1^{(-)}|$, in (4.1) is bounded by,

$$|\delta_1^{(-)}| \leq \frac{4}{7-p}\sqrt{\frac{2(8-p)}{p+1}} \tag{4.3}$$

for which the configuration remains real, and approaches zero (when $N \neq \bar{N}$ and $p \neq 3$) or the maximum value $\sqrt{2(8-p)/(7-p)}$ less than the bounded one given in eq. (4.3) (when $N = \bar{N}$ or $p = 3$) as $T \to \pi/2$. Note that in ref. [5], for simplicity, c_p was taken as $c = \sqrt{(7-p)/2(8-p)}$. Now since δ_1 was obtained from a quadratic relation (2.9) (with $q = 0$) it actually has two roots and we just kept the above bounded root there. This corresponds to one branch value of δ_1 and the other branch value of δ_1 is given by the other root as

$$\delta_1^{(+)} = \frac{1}{2c_p}\frac{a}{|a|}\left\{|a|\sqrt{\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}} \right. \\ \left. +\sqrt{a^2\left(\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}\right) + 4\left(\frac{2(8-p)}{7-p}c_p^2 - \cos^2 T\right)}\right\} \tag{4.4}$$

which, on the other hand, exceeds the bounded value in general, and for this reason it was dropped in ref. [5]. We know that the first expression gives a good description of the brane-antibrane annihilation leading to either a BPS Dp-brane or Minkowski vacuum i.e. the open string tachyon condensation. From our previous discussion the second expression will be responsible for the unbounded δ_1 and therefore may be related to a possible phase transition. Further, as will be shown, $\delta_1 = -12/7$ for the usual black 0-brane is a solution of $\delta_1^{(+)}$ but not $\delta_1^{(-)}$ in general except for a degenerate case.[6] This seems also consistent

[6] As will be clear from the following discussion, the two branches are disjoint when $N \neq \bar{N}$, otherwise they will meet at a point at $T = 0$, i.e., at the top of the tachyon potential as claimed in the introduction. One can check this explicitly by noting that in order to have $\delta_1^{(-)} = \delta_1^{(+)}$, the second square root in each expression must vanish which can occur only as $N = \bar{N}$ and c_p takes its minimal value given in (4.2).

with the possible phase transition for black branes through the closed string tachyon condensation to bubbles of nothing as proposed by Horowitz and others [14, 15]. Since we propose black brane to appear in some branch of the parameter space and since only for $p = 0$, the non-susy p-brane solution (which is obtained from (2.8) by putting $q = 0$ as well) can be mapped to black D0-brane so we will use $p = 0$ in (4.1) and (4.4). Now we will examine explicitly which of the above δ_1 eq. (4.1) or (4.4) will satisfy the condition for the black 0-brane i.e. $\delta_1 = -12/7$ (see eq. (3.15)) for certain value of the tachyon. To discuss this let us first set $N = \bar{N}$, i.e., the chargeless case ($N \neq \bar{N}$ will be discussed later). Since $a = -3/2$ for 0-brane, putting $\delta_1^{(-)} = -12/7$ in (4.1) we get

$$\frac{24c_0}{7} - \frac{3}{2}\cos T = -\sqrt{\frac{9}{4}\cos^2 T + 4\left(\frac{16}{7}c_0^2 - \cos^2 T\right)} \tag{4.5}$$

Now from eq. (4.2) we have $c_0 \geq 7/16$, the lhs of eq. (4.5) gives $24c_0/7 - 3\cos T/2 \geq (3/2)(1 - \cos T) \geq 0$. So, in general this equation can not be satisfied. The only way it can be satisfied is if the rhs of (4.5) vanishes. It can be easily checked that for this to happen c_0 must take the minimum value $7/16$ and $\cos T = 1$, i.e. the tachyon is at the top of the potential, a sort of critical value. For this c_0, we have from (4.1)

$$\delta_1^{(-)} = -\frac{4}{7}\left(3\cos T - \sqrt{7}\sin T\right) \tag{4.6}$$

We thus see from this expression that $\delta_1^{(-)} = -12/7$ is the smallest value and any other value of $\delta_1^{(-)}$ in the condensation will be larger. The implication of this is that just as the brane-antibrane starts to annihilate each other, the system has a choice to either form a regular horizon (then the brane-antibrane can no longer annihilate each other) or to annihilate each other without forming a regular horizon. In the former case an analog of a transition to "bubble of nothing"[7] may take over while for the latter it is just the open string tachyon condensation process. For all other values of c_0, we have only brane-antibrane annihilation via open string tachyon condensation and there is no regular horizon formation in this branch since $\delta_1^{(-)} = -12/7$ can never be satisfied.

Now we consider the second expression (4.4) for the black brane and in this case we get for $\delta_1^{(+)} = -12/7$,

$$\frac{24c_0}{7} - \frac{3}{2}\cos T = \sqrt{\frac{9}{4}\cos^2 T + 4\left(\frac{16}{7}c_0^2 - \cos^2 T\right)}. \tag{4.7}$$

This equation can be solved with either $\cos T = 16c_0/7$ or $\cos T = 2c_0/7$. The first solution can only be satisfied if $c_0 = 7/16$ (since this is the minimum value of c_0) and $\cos T = 1$ which is the critical case as mentioned above but there is some complication here. Let us discuss this in a bit detail. For $c_0 = 7/16$ we have from (4.4)

$$\delta_1^{(+)} = -\frac{4}{7}\left(3\cos T + \sqrt{7}\sin T\right). \tag{4.8}$$

[7]We will address the possible transition for 0-brane case in a forthcoming paper as will be mentioned later in this section.

It is not difficult to check that this $\delta_1^{(+)}$ has the smallest value of $-16/7$ at $\cos T = 3/4$, i.e. reaching its bounded value, and the largest value of $-4/\sqrt{7}$ at $\cos T = 0$ or $T = \pi/2$. Since $-4/\sqrt{7} > -12/7$, we therefore expect another solution for $\delta_1^{(+)} = -12/7$ between $\cos T = 3/4$ and $\cos T = 0$ in addition to the $\cos T = 1$. Indeed one can find that this occurs at $\cos T = 1/8$ from the above equation which is just given by $\cos T = 2c_0/7$ for $c_0 = 7/16$. Given that $|\delta_1^{(+)}| \leq 16/7$, i.e., its bounded value, for $0 \leq T \leq \pi/2$, we therefore expect that this $\delta_1^{(+)}$ for the chargeless case can also be used to describe the brane-antibrane annihilation or the open string tachyon condensation. The actual process may behave like this: just as the brane-antibrane starts to annihilate each other, the system has a choice either to form a regular horizon (then the brane-antibrane can no longer annihilate each other and the possible phase transition associated with the black brane is taken over) or to annihilate each other without forming a regular horizon. When the system makes a choice of annihilating each other initially, i.e., at $\cos T = 1$, it will continue so until $\cos T = 1/8$. At this point, the system has again a choice either to form a regular horizon without continuing the annihilation or to continue the annihilation to end up with a Minkowski vacuum. We now consider $7/16 < c_0 \leq 7/2$. The only solution to (4.7) is $\cos T = 2c_0/7$ as given above. We now have from (4.4)

$$\delta_1^{(+)} = -\frac{1}{2c_0}\left(\frac{3}{2}\cos T + \sqrt{\frac{64}{7}c_0^2 - \frac{7}{4}\cos^2 T}\right). \tag{4.9}$$

The $\delta_1^{(+)}$ has a minimum value of $-16/7$, which is also its bounded value, at $\cos T = 12c_0/7$ if $7/16 < c_0 \leq 7/12$. Given that there is only one solution $\cos T = 2c_0/7$ now and $|\delta_1^{(+)}| \leq 16/7$, we therefore expect the following process: the brane-antibrane system initially annihilates each other until $\cos T = 2c_0/7$. At this point, it has a choice either to form a regular horizon without annihilating each other further or to continue to annihilate each other without forming a regular horizon to end up with a Minkowski vacuum.

Let us next discuss the case when $N \neq \bar{N}$. For this case the condition for the black brane $\delta_1 = -12/7$ gives from the first expression (4.1),

$$\frac{24c_0}{7} = \frac{3}{2}\sqrt{\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}} - \sqrt{\frac{9}{4}\left(\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}\right) + 4\left(\frac{16}{7}c_0^2 - \cos^2 T\right)}. \tag{4.10}$$

We have to see whether this equation can be made consistent for the allowed values of c_0 and some values of $\cos T$. In order for eq. (4.10) to be consistent, we need to have the second term on the right smaller than the first term and this can only be true if the second term in the square root is negative, i.e.,

$$\frac{16}{7}c_0^2 - \cos^2 T < 0 \tag{4.11}$$

or $0 < c_0 < \sqrt{7}\cos T/4$. But actually for consistency we have to have the following equation to be satisfied (which can be derived from (4.10))

$$\frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T} = \frac{(\frac{16}{7}c_0 - \cos T)(\frac{2}{7}c_0 - \cos T)(\frac{16}{7}c_0 + \cos T)(\frac{2}{7}c_0 + \cos T)}{(\frac{18}{7}c_0)^2} \tag{4.12}$$

The left side of the above equation is positive and so must be the right side. Given that $c_0 < (\sqrt{7}/4)\cos T$, i.e., $2c_0/7 < (1/2\sqrt{7})\cos T < \cos T$, we must have from (4.12)

$$\frac{16}{7}c_0 - \cos T < 0 \tag{4.13}$$

But the condition (4.13) gives $c_0 < (7/16)\cos T$ which is even more constrained than the bound $c_0 < (\sqrt{7}/4)\cos T$. In other words, for consistency we must have this constraint to be satisfied but this one contradicts with the previous constraint $c_0 \geq 7/16$ derived earlier. Therefore we can not make (4.10) consistent implying that there is no solution from the expression (4.1) for $\delta_1^{(-)}$ to form a regular horizon which is the expected result. For the second expression (4.4) for $\delta_1^{(+)}$, we expect in general a solution since the relevant equation is

$$\frac{24c_0}{7} = \frac{3}{2}\sqrt{\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}} + \sqrt{\frac{9}{4}\left(\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}\right) + 4\left(\frac{16}{7}c_0^2 - \cos^2 T\right)} \tag{4.14}$$

Now we still have $c_0 \geq 7/16$ but here we don't need to have $c_0 < (\sqrt{7}/4)\cos T$ as before for consistency. But we still have the equation (4.12) to be satisfied. So the condition for the right side to be positive and consistent with $c_0 \geq 7/16$ is $c_0 > 7/2$. Since the term in the square root on the right side of (4.14) is $(64/7)c_0^2$ whose square root is less than the left side of this equation, therefore we are certain that there must be at least a solution of this equation for $\cos T$ as expected. Moreover from (4.4) for $p = 0$, $|\delta_1^{(+)}|$ for the charged case will be unbounded for small $\cos T$ and therefore we expect that this branch of $\delta_1^{(+)}$ will be responsible for possible phase transition related to the black brane such as from black brane to "bubble of nothing".

(b) Intersecting D4/$\bar{\text{D}}$4 with chargeless D0/$\bar{\text{D}}$0 system. The supergravity solutions of intersecting Dp/$\bar{\text{D}}p$ with D$(p-4)$/$\bar{\text{D}}(p-4)$ system where both brane-antibrane systems are charged are given in ref. [6]. These solutions are characterized by five parameters ω, θ_1, θ_2, $\Delta_1(=\alpha_1+\beta_1)$ and $\Delta_2(=\alpha_2+\beta_2)$. We would like to refer the reader to [6] for the details of the solution, but we just mention that the subscript 1 and 2 refer to Dp/$\bar{\text{D}}p$ system and D$(p-4)$/$\bar{\text{D}}(p-4)$ system respectively. These supergravity parameters were related in [6] to physical microscopic parameters, namely, the number of Dp-branes (N_1), the number of $\bar{\text{D}}p$-branes ($\bar{N}_1$), the number of D$(p-4)$-branes (N_2), the number of $\bar{\text{D}}(p-4)$-branes ($\bar{N}_2$) and the tachyon parameter (T) such that we get the correct picture of brane-antibrane annihilation via tachyon condensation. These relations are not known for other intersecting brane solutions. Since the black-brane appears when one of the intersecting branes is the chargeless 0-brane/anti-0-brane, we will make use of the solutions of ref. [6] only for $p = 4$ after discussing some generalities. From our experience of tachyon condensation on the simple Dp/$\bar{\text{D}}p$ system and the fact that there is no interaction between the Dp/$\bar{\text{D}}p$ and D$(p-4)$/$\bar{\text{D}}(p-4)$ system we found in (see eq. (2.10) of [6]) that the parameters Δ_1 and

Δ_2 must satisfy,

$$\Delta_1\sqrt{1+\frac{(N_1-\bar{N}_1)^2}{c_p^2A^2\Delta_1^2\cos^2T}} = \pm\left[\frac{1}{c_pA}\sqrt{\frac{(N_1-\bar{N}_1)^2}{\cos^2T}+4N_1\bar{N}_1\cos^2T}-\frac{p-3}{2}\delta\right]$$
$$\Delta_2\sqrt{1+\frac{a^2(N_2-\bar{N}_2)^2}{c_p^2A^2\Delta_2^2\cos^2T}} = \pm\left[\frac{a}{c_pA}\sqrt{\frac{(N_2-\bar{N}_2)^2}{\cos^2T}+4N_2\bar{N}_2\cos^2T}-\frac{p-7}{2}\delta\right] \quad (4.15)$$

where in the above $A=\sqrt{N_1\bar{N}_1}+a\sqrt{N_2\bar{N}_2}$ and $a=(2\pi\sqrt{\alpha'})^4/V_4$ with V_4 the volume of the compact directions. Now using the parameter relation analogous to (2.9) or (2.14) in that case given by,

$$\Delta_1^2+\Delta_2^2+\left(4-\frac{(p-3)^2}{4}-\frac{(p-7)^2}{4}\right)\delta^2=8\frac{8-p}{7-p} \quad (4.16)$$

we obtain a quadratic relation in δ, after substituting Δ_1 and Δ_2 from (4.15) in (4.16). So, the equation has two roots given by,

$$\delta^{(-)} = \frac{1}{8c_pA}\frac{K}{|K|}\left[|K|-\sqrt{K^2+64\left(\frac{2(8-p)}{7-p}c_p^2A^2-\left(N_1\bar{N}_1+a^2N_2\bar{N}_2\right)\cos^2T\right)}\right]$$
$$\delta^{(+)} = \frac{1}{8c_pA}\frac{K}{|K|}\left[|K|+\sqrt{K^2+64\left(\frac{2(8-p)}{7-p}c_p^2A^2-\left(N_1\bar{N}_1+a^2N_2\bar{N}_2\right)\cos^2T\right)}\right] \quad (4.17)$$

where,

$$K=(p-3)\sqrt{\frac{(N_1-\bar{N}_1)^2}{\cos^2T}+4N_1\bar{N}_1\cos^2T}+a(p-7)\sqrt{\frac{(N_2-\bar{N}_2)^2}{\cos^2T}+4N_2\bar{N}_2\cos^2T} \quad (4.18)$$

In (4.17) $\delta^{(-)}$ is bounded while $\delta^{(+)}$ is not when $\cos T$ approaches zero and so only the first one is responsible for the open string tachyon condensation when either $N_1\neq\bar{N}_1$ or $N_2\neq\bar{N}_2$ or both, i.e., the charged case. Let us point out some features of the solutions (4.17). First, the term in the square root should be positive for reality of the solutions. This in general sets the bound for c_p. Second, in order to get black Dp-brane, we know from (3.12) that $\delta=-2(6-p)/(7-p)<0$ and this implies that we need $K<0$ for at least some values of $\cos T$. It is obvious that when $K>0$, the second equation in (4.17) can not be satisfied for the negative δ, but this is not obvious from the first one. We will show that even for this case δ can not be negative, therefore no solution exists for this δ when $K>0$.

Let us now put $p=4$ and for our purpose of making connection with black D4-brane we put $N_2=\bar{N}_2$. Let us also for simplicity consider first the case of $N_1=\bar{N}_1$ i.e. the chargeless D4/$\bar{\text{D}}$4 system. Unlike the D0-case discussed previously, here we need to fix both $\Delta_1(=\alpha_1+\beta_1)$ and $\Delta_2(=\alpha_2+\beta_2)$ to be 2 and also from (3.12) $\delta=-4/3$. Indeed we note from the general system given in (3.1) and (3.2) that we have $\Delta_1=\alpha_1+\beta_1=2$ and $\Delta_2\equiv\alpha_2=2$ and from (3.12) $\delta=-4/3$. Now by using the first equation of (4.15) for the present case, we get

$$\frac{N_1}{c_4A}\cos T=\frac{2}{3} \quad (4.19)$$

Note that we have used the plus sign of the rhs of the equation since with minus sign this equation can not be satisfied. On the other hand from the second equation of (4.15) we get,

$$\frac{aN_2}{c_4 A}\cos T = 2 \tag{4.20}$$

Here also plus sign is chosen as with the minus sign the equation can not be satisfied except for some degenerate case which we ignore. These two imply that the solution exists only if $aN_2 = 3N_1$ and now $\cos T = 8c_4/3$. Note that we have $A = 4N_1$ and $K = 2(N_1 - 3aN_2)\cos T = -16N_1 \cos T < 0$ and this is consistent with our above observation. Now in order for δ's to remain real (i.e. for the quantity inside the square root of eq. (4.17) to remain positive) we have from (4.17)

$$c_4^2 \geq \frac{3}{8A^2}\left[\left(N_1^2 + a^2 N_2^2\right) - \frac{(N_1 - 3aN_2)^2}{16}\right]\cos^2 T \tag{4.21}$$

Since c_4 is a constant, independent of the N's and the value of $\cos T$, we need to have the following condition so that (4.21) can always be satisfied,

$$c_4^2 \geq \frac{3}{8A^2}\left[\left(N_1^2 + a^2 N_2^2\right) - \frac{(N_1 - 3aN_2)^2}{16}\right] \tag{4.22}$$

Now let us see what is the value of c_4 for the present solution where $aN_2 = 3N_1$. We find from (4.22) $c_4 \geq 3/8$. Now we have to see whether this condition is consistent with the solution namely, $\cos T = 8c_4/3$. It is obvious that this condition is consistent only for the critical value $c_4 = 3/8$ and $\cos T = 1$. Actually for this case the square root in (4.17) vanishes and so solves both the equations.[8] The explanation for the underlying picture is the same as for the D0-case i.e. the system starts at the top of the potential and it has the choice of either forming the brane-antibranes and subsequently annihilate each other through open string tachyon condensation to end up with a Minkowski vacuum or forming the regular horizon for the black D4 phase and subsequently decay to bubbles through possible closed string tachyon condensation.

Let us next discuss the general case where $N_1 \neq \bar{N}_1$. It is in general quite involved to discuss the solution directly from (4.17) and much easier to use eq. (4.15) with $p = 4$ instead. The second equation in this case has exactly the same form as before

$$aN_2 \cos T(aN_2 \cos T - 2c_4 A) = 0 \tag{4.23}$$

where we have used $\Delta_2 = 2$, and $\delta = -4/3$. Note that $\cos T \neq 0$ comparing the same $\cos T$ obtained from the first equation in (4.15). Now since $(p-3)\delta/2 = -2/3 < 0$, only the plus sign can be taken in the first equation of (4.15) and the equation is,

$$\sqrt{4 + \frac{(N_1 - \bar{N}_1)^2}{c_4^2 A^2 \cos^2 T}} - \frac{2}{3} = \sqrt{4\frac{4N_1\bar{N}_1}{a^2 N_2^2} + \frac{(N_1 - \bar{N}_1)^2}{c_4^2 A^2 \cos^2 T}} \tag{4.24}$$

[8]Again we see a meet point at $\cos T = 1$ of the two branches for the chargeless case.

where we have used $aN_2 \cos T = 2c_4 A$ from eq. (4.23) in some of the terms above. It is clear from (4.24) that in order to have a consistent equation we must have the following constraint,

$$\frac{4N_1\bar{N}_1}{a^2N_2^2} < 1 \tag{4.25}$$

Further from the same equation (4.24) we also have,

$$\frac{a^2N_2^2(N_1-\bar{N}_1)^2}{16(c_4^2A^2)^2} = \left(\frac{2}{3} - \frac{6N_1\bar{N}_1}{a^2N_2^2}\right)\left(\frac{8}{3} - \frac{6N_1\bar{N}_1}{a^2N_2^2}\right) \tag{4.26}$$

where the positivity of the lhs of the above equation gives further, by considering (4.25),

$$\frac{4N_1\bar{N}_1}{a^2N_2^2} < \frac{4}{9} \tag{4.27}$$

and also we solve from (4.24) the following,[9]

$$\begin{aligned}
\cos^2 T &= \frac{|N_1-\bar{N}_1|}{2aN_2\sqrt{(1/3-3N_1\bar{N}_1/a^2N_2^2)(4/3-3N_1\bar{N}_1/a^2N_2^2)}}, \\
c_4^2A^2 &= \frac{aN_2|N_1-\bar{N}_1|}{8\sqrt{(1/3-3N_1\bar{N}_1/a^2N_2^2)(4/3-3N_1\bar{N}_1/a^2N_2^2)}},
\end{aligned} \tag{4.28}$$

where we have also used $\cos T = 2c_4A/aN_2$ from eq. (4.23). In obtaining (4.28), we have squared some equations and have not checked the consistencies that the term inside the square root in (4.17) remains positive. We need to check a few things before we can be claim that the above solution indeed exists. Let us discuss under what conditions, a solution with $\delta < 0$ can exist. Let us first check the value of K in (4.18). In this case we have

$$K = \sqrt{\frac{(N_1-\bar{N}_1)^2}{\cos^2 T} + 4N_1\bar{N}_1\cos^2 T} - 6aN_2\cos T \tag{4.29}$$

We see that K is positively infinite when $\cos T \to 0$. So if the minimum value of K is positive, then K is always positive. If this happens, then we expect a possible solution from the first equation in (4.17) only when

$$\frac{8}{3}c_4^2A^2 \geq \left(N_1\bar{N}_1 + a^2N_2^2\right)\cos^2 T \tag{4.30}$$

When $K < 0$ for certain values of $\cos T$, then a solution is possible from the first equation when the above inequality is reversed and from the second equation of (4.17) in general. So we now check the sign for K from the solution (4.28). Using this in (4.18) we get

$$K = -\frac{\sqrt{2aN_2|N_1-\bar{N}_1|}}{\left[(1/3-3N_1\bar{N}_1/a^2N_2^2)(4/3-3N_1\bar{N}_1/a^2N_2^2)\right]^{1/4}}\left(\frac{7}{3} + \frac{3N_1\bar{N}_1}{a^2N_2^2}\right) < 0 \tag{4.31}$$

[9]Note that the brane number N's and the parameter a appearing in (4.28) should be constrained such that $\cos^2 T \leq 1$ and also the constraint for c_4 is not violated.

This shows that (4.28) may be a solution. Before we make sure this and know which one in (4.17) actually solves this, let us evaluate the term with the square root in (4.17) and we find

$$\begin{aligned}&\sqrt{K^2+64\left[\frac{8}{3}c_4^2A^2-\left(N_1\bar{N}_1+a^2N_2^2\right)\cos^2 T\right]}\\&=\frac{\sqrt{2aN_2|N_1-\bar{N}_1|}}{\left[(1/3-3N_1\bar{N}_1/a^2N_2^2)(4/3-3N_1\bar{N}_1/a^2N_2^2)\right]^{1/4}}\left(\frac{1}{3}-\frac{3N_1\bar{N}_1}{a^2N_2^2}\right)\end{aligned}\tag{4.32}$$

where we made use of (4.27). From (4.31) and (4.32) one can check directly that the solution in (4.28) is indeed a solution of the second equation in (4.17) but not the first one. This is entirely consistent with our expectation since now $\cos T \neq 1$ and only the unbounded channel (or the second solution of (4.17)) is associated with the regular horizon formation as we conjectured.

As a closing remark in this section we emphasize that in this paper we have made use of the semi-empirical relations, obtained previously in refs. [5, 6], between the SUGRA parameters and microscopic physical parameters (number of branes, number of antibranes and the open string tachyon parameter) of the (intersecting) non-susy brane system to discuss the brane-antibrane annihilation process and the black brane formation and the related phase transition. In particular, for the charged system we have shown that the specific value of the parameter δ for the black brane can only occur in the unbounded channel. Given that the non-susy brane configuration represents the underlying brane-antibrane system and in general $\cos T \neq 1$ for the black brane value of δ in the second expression, we can expect that the brane-antibrane in this case starts to annihilate from $\cos T = 1$ to the value of $\cos T$ corresponding to the black brane value of δ. At this particular value of $\cos T$, the system has a choice of either forming a regular horizon (the possible closed string tachyon condensation will be taken over as discussed by Horowitz to end up with a bubble of nothing) or continuing the annihilation so that δ reaches its bounded value. Then also a possible phase transition exists since the configuration becomes complex there once δ exceeds its bounded value and this is corroborated by the fact that right at the bounded value of δ, the corresponding configuration also has a horizon which is null-singular. We are currently investigating the possible phase transitions related to the different dynamics considered here. However, Horowitz's picture of closed string tachyon condensation of black brane doesn't seem to apply for the $p = 0$ case and and it appears that we need to lift the configuration to eleven dimensions. Whether our current investigation for $p = 0$ case can be of any help in understanding the various phase transitions we mentioned is to be seen.

5. Conclusion

To conclude, contrary to the common belief, we have argued in this paper that the non-susy (non-extremal) p-branes are of only one type and not of two types. Usually it is known that there are two possible ways of constructing the non-susy p-branes in type II supergravities

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and they are referred to as type 1 and type 2 in the literature [11]. The type 1 metric has the form

$$ds^2 = e^{2A}\left(dr^2 + r^2 d\Omega_{8-p}^2\right) + e^{2B}\left(-dt^2 + \sum_{i=1}^{p}(dx^i)^2\right) \tag{5.1}$$

in ten dimensions, where the functions A and B are quite general but depends on r only. For the type 1 solution A, B satisfy

$$(p+1)B + (7-p)A = \ln G \tag{5.2}$$

where $G(r)$ is an arbitrary function. Note that when $G(r) = 1$, the type 1 solution reduces to the usual BPS p-brane solution. In ref. [1] this solution is referred to as non-susy p-brane solution and has been interpreted as p-brane/anti-p-brane (or non-BPS p-brane) solution. On the other hand the type 2 metric has the form,

$$ds^2 = e^{2A}\left(f^{-1}\, dr^2 + r^2 d\Omega_{8-p}^2\right) + e^{2B}\left(-f\, dt^2 + \sum_{i=1}^{p}(dx^i)^2\right) \tag{5.3}$$

and now the function A and B satisfy

$$(p+1)B + (7-p)A = 0 \tag{5.4}$$

just like a BPS p-brane. We have shown that type 2 solutions are in fact contained in the generalized type 1 as a special case and therefore we only have one type of solutions, namely, the type 1 solutions.

To show this we know that non-susy branes of type 1 can be of different kinds. That is, there can be only one kind of brane involved or there can be intersections of many kinds of branes, where the different branes can be either charged or chargeless under various form-fields. We have argued that the type 2 p-brane solution can be regarded as intersecting non-susy p-brane with chargeless non-susy 0-brane. We have shown this explicitly when the type 2 solution is the usual black p-brane. So, we first constructed an intersecting solution of non-susy p-brane with chargeless non-susy q-brane i.e. of type 1. This is constructed from a non-susy q-brane solution delocalized in $(p-q)$ transverse directions and then applying T-dualities successively in all the delocalized directions. The resulting solution has $(p-q+3)$ independent parameters and is anisotropic in the delocalized directions. When we made the solution isotropic (in the delocalized directions) by setting the parameters associated with these directions equal, the solution has an isometry group ISO$(1,q)$ $\times$ SO$(p-q)$ $\times$ SO$(9-p)$ and is dependent on four independent parameters. From this isometry we recognized the solution to be intersecting non-susy p-brane with chargeless non-susy q-brane. The non-susy q-brane is chargeless because there is no gauge field associated with it, unlike the non-susy p-brane. The intersecting solution when both the branes are charged is known explicitly only when $p-q=4$ or 0. We have compared these solutions with the ones we obtained in this paper. Next we showed that when we set $q=0$ i.e. when the isometry group becomes R $\times$ ISO(p) $\times$ SO$(9-p)$, the resulting solution can be mapped to the standard black p-brane solutions by a coordinate transformation when two of the four parameters (ω, θ, δ, α_2 or ω, θ, δ_1, δ_2) take some special ($\delta = -2(6-p)/(7-p)$, $\alpha_2 = 2$ or $\delta_1 = -12/7$, $\delta_2 = -1/7$) values. This is expected since the black brane has two parameters corresponding to its mass and the charge. This in turn shows that the black branes are indeed of type 1 or intersecting non-susy p-brane with q-brane solution of special kind.

(Intersecting) non-susy branes of type 1 has a natural interpretation as (intersecting) brane-antibrane systems. Since there are tachyons in the world-volume of these unstable systems, they usually decay to BPS branes or Minkowski vacuum depending on the initial charge of the system. This decay process (or the tachyon condensation) can be described by relating the various SUGRA parameters appearing in the solution semi-empirically to the microscopic physical parameters of the system. But in this process usually we do not encounter the formation of a regular formation. We have demonstrated that this is because there exists two disjoint processes, namely, the brane-antibrane annihilation process and the black-brane related phase transition process in the parameter space of non-susy branes. We have shown this by giving two concrete examples, namely the D0/$\bar{\text{D}}$0 system and the intersecting D4/$\bar{\text{D}}$4 and chargeless D0/$\bar{\text{D}}$0 system where the explicit relations between the SUGRA parameters and the microscopic physical parameters are known. The brane-antibrane annihilation process where the underlying dynamics is governed by the open string tachyon condensation is much better understood and has been studied in specific cases in refs. [5, 6]. The other process, i.e., the regular horizon formation and the subsequent evolution or transition to non-susy bubbles where the underlying dynamics is governed by the possible closed string tachyon condensation except for the $p = 0$ case has been discussed by Horowitz and others [14, 15]. The $p = 0$ case as well as other issues as mentioned in the text are under current investigation and we hope to come back to these in a future publication.

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References

[1] J.X. Lu and S. Roy, *Static, non-SUSY p-branes in diverse dimensions*, *JHEP* **02** (2005) 001 [hep-th/0408242].

[2] B. Zhou and C.-J. Zhu, *The complete black brane solutions in D-dimensional coupled gravity system*, hep-th/9905146.

[3] V.D. Ivashchuk and V.N. Melnikov, *Exact solutions in multidimensional gravity with antisymmetric forms*, *Class. and Quant. Grav.* **18** (2001) R87 [hep-th/0110274].

[4] P. Brax, G. Mandal and Y. Oz, *Supergravity description of non-BPS branes*, *Phys. Rev.* **D 63** (2001) 064008 [hep-th/0005242].

[5] J.X. Lu and S. Roy, *Supergravity approach to tachyon condensation on the brane-antibrane system*, *Phys. Lett.* **B 599** (2004) 313 [hep-th/0403147].

[6] H. Bai, J.X. Lu and S. Roy, *Tachyon condensation on the intersecting brane-antibrane system*, *JHEP* **08** (2005) 068 [hep-th/0506115].

[7] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[8] G.T. Horowitz and A. Strominger, *Black strings and p-branes*, *Nucl. Phys.* **B 360** (1991) 197.

[9] M.J. Duff and J.X. Lu, *Black and super p-branes in diverse dimensions*, *Nucl. Phys.* **B 416** (1994) 301 [hep-th/9306052].

[10] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons*, *Phys. Rept.* **259** (1995) 213 [hep-th/9412184].

[11] H. Lu, S. Mukherji and C.N. Pope, *From p-branes to cosmology*, *Int. J. Mod. Phys.* **A 14** (1999) 4121 [hep-th/9612224].

[12] Y.-G. Miao and N. Ohta, *Complete intersecting non-extreme p-branes*, *Phys. Lett.* **B 594** (2004) 218 [hep-th/0404082].

[13] J.X. Lu and S. Roy, *Non-SUSY p-branes delocalized in two directions, tachyon condensation and T-duality*, *JHEP* **06** (2005) 026 [hep-th/0503007].

[14] G.T. Horowitz, *Tachyon condensation and black strings*, *JHEP* **08** (2005) 091 [hep-th/0506166].

[15] G.T. Horowitz and E. Silverstein, *The inside story: quasilocal tachyons and black holes*, *Phys. Rev.* **D 73** (2006) 064016 [hep-th/0601032].

[16] J.X. Lu and S. Roy, *Fundamental strings and NS5-branes from unstable D-branes in supergravity*, *Phys. Lett.* **B 637** (2006) 326 [hep-th/0508045].

[17] E. Bergshoeff, C.M. Hull and T. Ortin, *Duality in the type-II superstring effective action*, *Nucl. Phys.* **B 451** (1995) 547 [hep-th/9504081].

[18] A.K. Das and S. Roy, *On M-theory and the symmetries of type-II string effective actions*, *Nucl. Phys.* **B 482** (1996) 119 [hep-th/9605073].

[19] A. Sen, *Tachyon dynamics in open string theory*, *Int. J. Mod. Phys.* **A 20** (2005) 5513 [hep-th/0410103].

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